

**St Christopher School**

**VIth Form Entrance Paper**

Please attempt all questions.

Marks are awarded for workings.

Answers with no mathematical explanation or proof will not be awarded full marks.

Calculators are required for the test.

Time Allowed 90 minutes

*For the Examiner's Use Only*

|          |  |     |  |
|----------|--|-----|--|
| 1        |  | 6   |  |
| 2        |  | 7   |  |
| 3        |  | 5   |  |
| 4        |  | 8   |  |
| 5        |  | 6   |  |
| 6        |  | 7   |  |
| 7        |  | 10  |  |
| 8        |  | 6   |  |
| 9        |  | 4   |  |
| 10       |  | 6   |  |
| 11       |  | 4   |  |
| 12       |  | 7   |  |
| 13       |  | 5   |  |
| 14       |  | 3   |  |
| 15       |  | 5   |  |
| 16       |  | 6   |  |
| 17       |  | 10  |  |
| $\Sigma$ |  | 105 |  |

1.

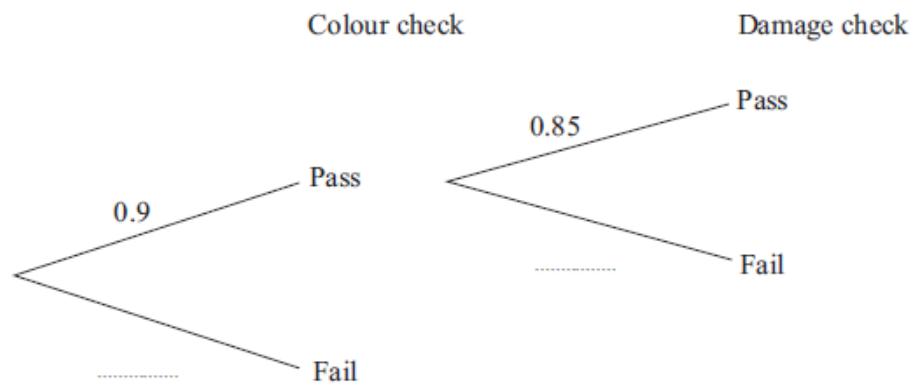
Joe has a job packing tomatoes in boxes.

First he has to check the colour of each tomato.

He also has to check the tomato for damage.

The decision tree diagram gives information about the probability of a tomato passing the colour check and the probability of a tomato passing the damage check.

(a) Complete the decision tree diagram.



(2)

Joe packs the tomatoes that pass both the colour check and the damage check.

On Thursday Joe has to check 9600 tomatoes.

(b) Work out an estimate for the number of these tomatoes he will pack.

.....  
(4)

2.

The diagram represents a door wedge.  
The door wedge is in the shape of a triangular prism.

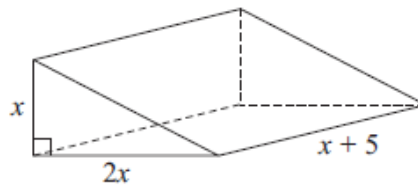


Diagram NOT  
accurately drawn

All the measurements shown on the diagram are in centimetres.

The triangular prism has a volume of  $100 \text{ cm}^3$ .

(a) Show that  $x^3 + 5x^2 = 100$

(2)

(b) Use trial and improvement to find the value of  $x$ .  
You must show all your working.  
Give your answer correct to 1 decimal place.

$x = \dots\dots\dots$   
(5)

3.

$AT$  is a vertical tower on horizontal ground as shown in the diagram.

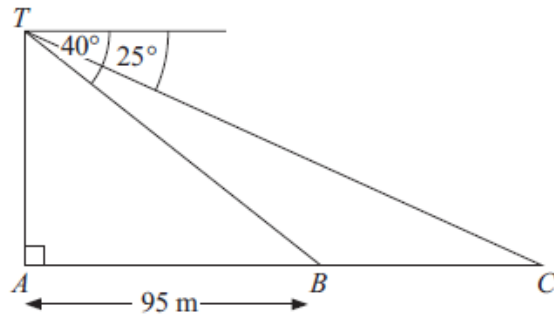


Diagram **NOT**  
accurately drawn

$ABC$  is a straight line.

$AB = 95\text{ m}$ .

The angle of depression of  $B$  from  $T$  is  $40^\circ$

The angle of depression of  $C$  from  $T$  is  $25^\circ$

Calculate the distance of  $B$  from  $C$ .

Give your answer correct to 3 significant figures.

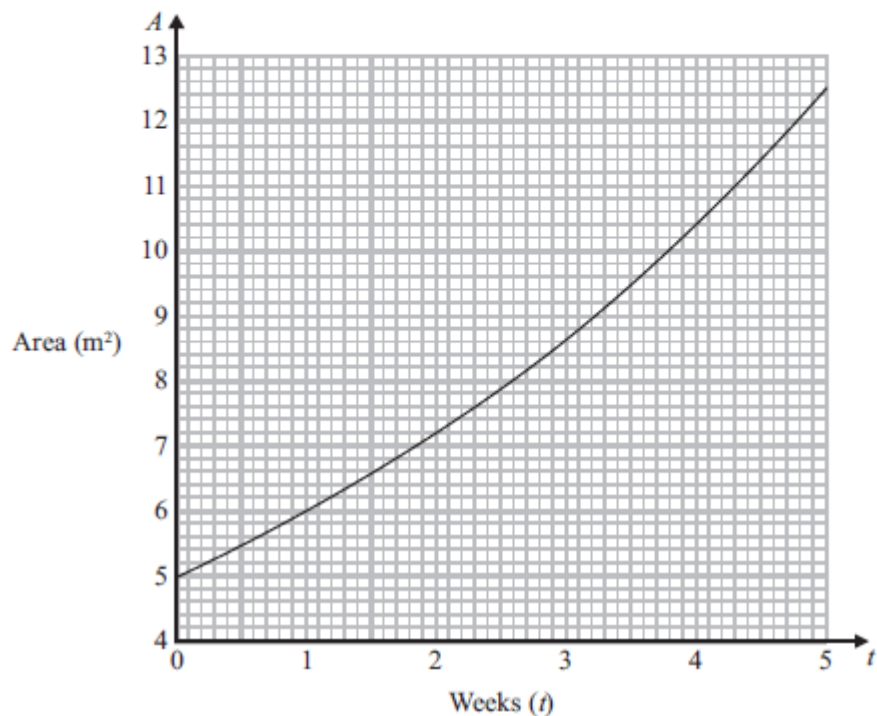
..... m

4.

Waterweed is growing in a pond.

Each week a student records the area of the pond covered by the waterweed.

The graph gives information about the area,  $A \text{ m}^2$ , covered by the waterweed after  $t$  weeks.



The area covered by the waterweed increased during the first 4 weeks.

(a) How much did the area increase during the first 4 weeks?

.....  $\text{m}^2$   
(2)

(b) The area,  $A \text{ m}^2$ , covered by the waterweed after  $t$  weeks is given by the formula  $A = ka^t$

(i) Write down the value of  $k$ .

.....

(ii) Work out an estimate for the value of  $a$ .

.....  
(3)

4 continued

Waterweed is also growing in a different pond.  
The student puts weed killer on this waterweed.

The area covered by the waterweed decreases by  $x\%$  each week.

The area covered by the waterweed halves in 10 weeks.

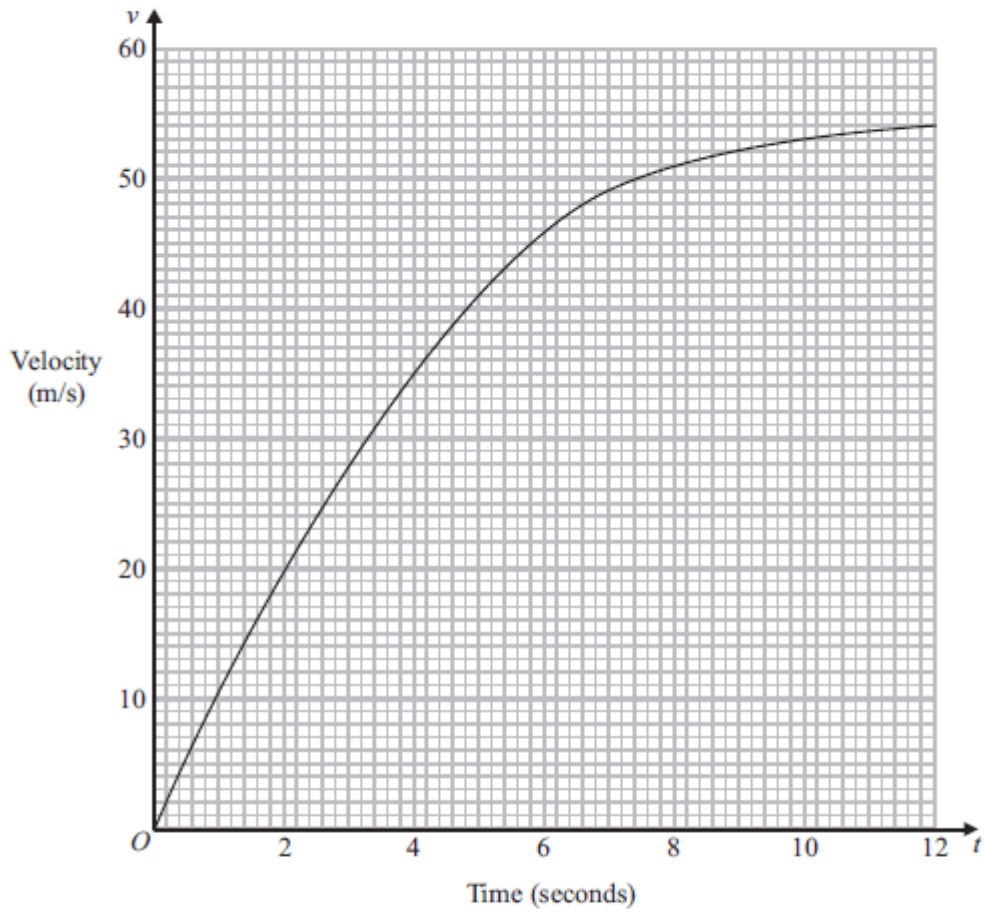
(c) Work out the value of  $x$ .

Give your answer correct to 1 decimal place.

5.

A parachutist jumps out of a plane.

This graph shows information about the velocity,  $v$  m/s, of the parachutist  $t$  seconds after he jumped.



(a) Work out an estimate for the acceleration of the parachutist when  $t = 8$

..... m/s<sup>2</sup>  
(3)

5 continued

(b) Work out an estimate for the distance the parachutist falls in the first 6 seconds.

..... m  
(3)



6.

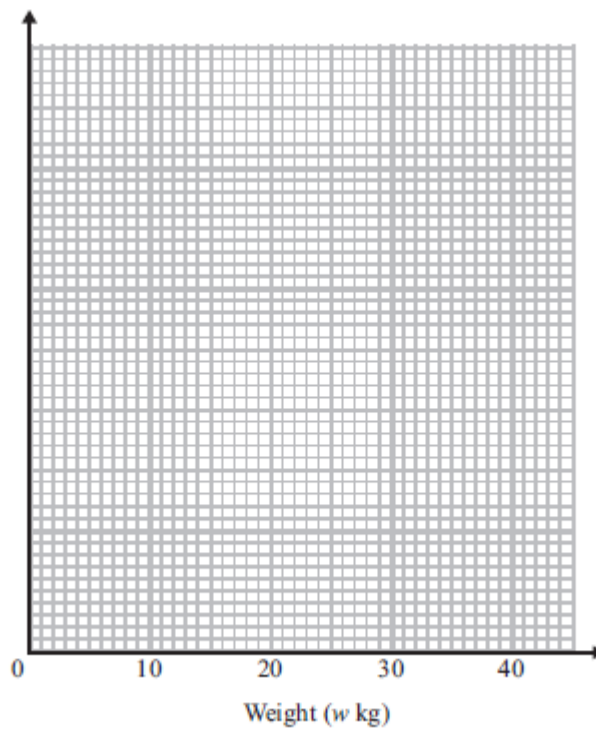
The table gives information about the weights, in kg, of some parcels.

| Weight ( $w$ kg) | Frequency |
|------------------|-----------|
| $0 < w \leq 5$   | 20        |
| $5 < w \leq 10$  | 13        |
| $10 < w \leq 20$ | 10        |
| $20 < w \leq 40$ | 7         |

(a) Work out an estimate for the mean weight of a parcel.

..... kg  
(4)

(b) On the grid, draw a histogram for the information in the table.



(3)

7.

Dennis is going to buy some bookcases and some lamps to sell in his shop.

Each bookcase costs £18

Each lamp costs £12

Dennis has a maximum of £360 to spend.

He is going to buy  $x$  bookcases.

He is going to buy  $y$  lamps.

(a) Show that  $3x + 2y \leq 60$

(2)

The first constraint is  $3x + 2y \leq 60$

The other constraints are  $y \leq 2x$ ,  $x + y \leq 24$

(b) On the grid, represent these constraints and show the feasible region.

(4)

Dennis makes a profit of £20 for each bookcase he sells.

He makes a profit of £15 for each lamp he sells.

(c) Write down the objective function.

(1)

(d) Use your objective function to find

- (i) the greatest profit Dennis can make,
- (ii) the number of bookcases he should buy to make the greatest profit,
- (iii) the number of lamps he should buy to make the greatest profit.

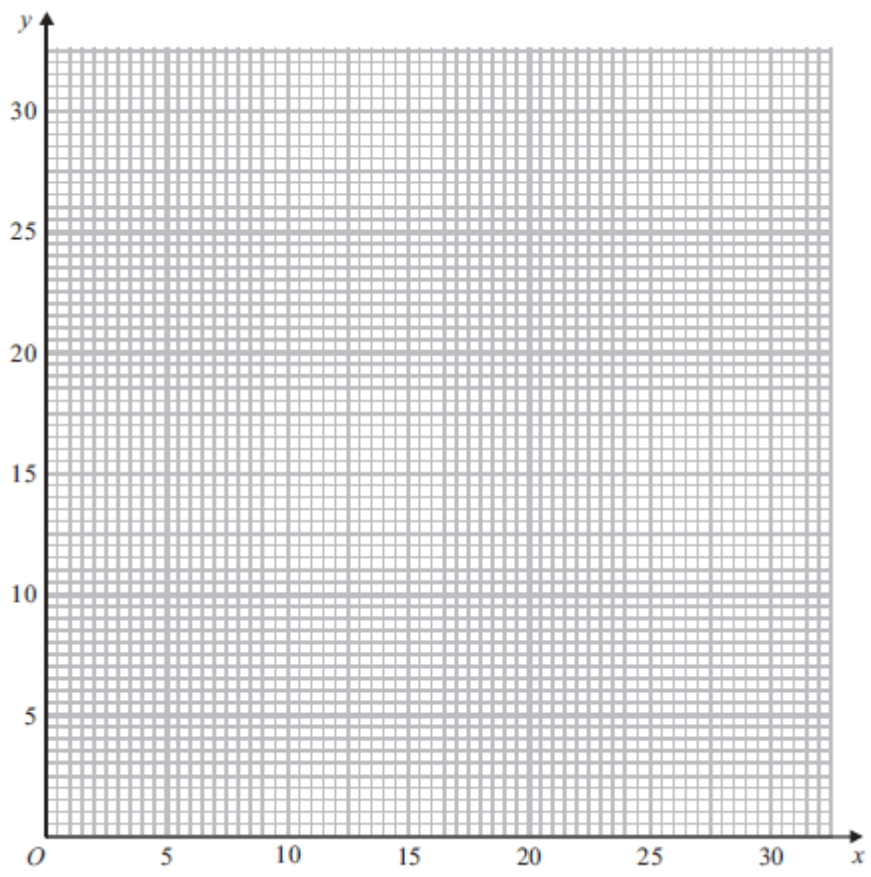
greatest profit £ .....

number of bookcases .....

number of lamps .....

(3)

7 continued



8.

A company makes two different desks.

The top of one desk is in the shape of a trapezium.

The top of the other desk is in the shape of a rectangle.

The diagram shows the tops of the two desks.

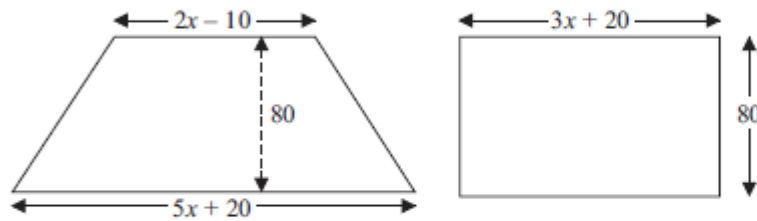


Diagram **NOT**  
accurately drawn

All measurements are in centimetres.

The tops of the two desks have the same area.

Work out the length, in centimetres, of the rectangular desk.  
You must show all your working.

..... cm

9.

Sal asked 60 adults if they liked Chinese food best or Italian food best or Thai food best.

29 of the adults were women.

6 of the women liked Thai food best.

10 of the men liked Chinese food best.

8 of the 13 adults who liked Italian food best were women.

Work out the number of men who liked Thai food best.

---

(4 marks)

10.

$ABC$  is an isosceles triangle.

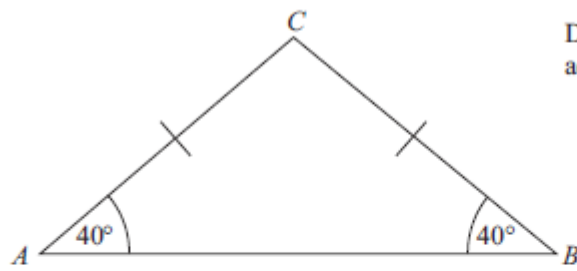


Diagram NOT  
accurately drawn

The area of this isosceles triangle is  $25 \text{ cm}^2$ .

Work out the length of each side of the triangle.  
Give your answers correct to 3 significant figures.

---

(6 marks)

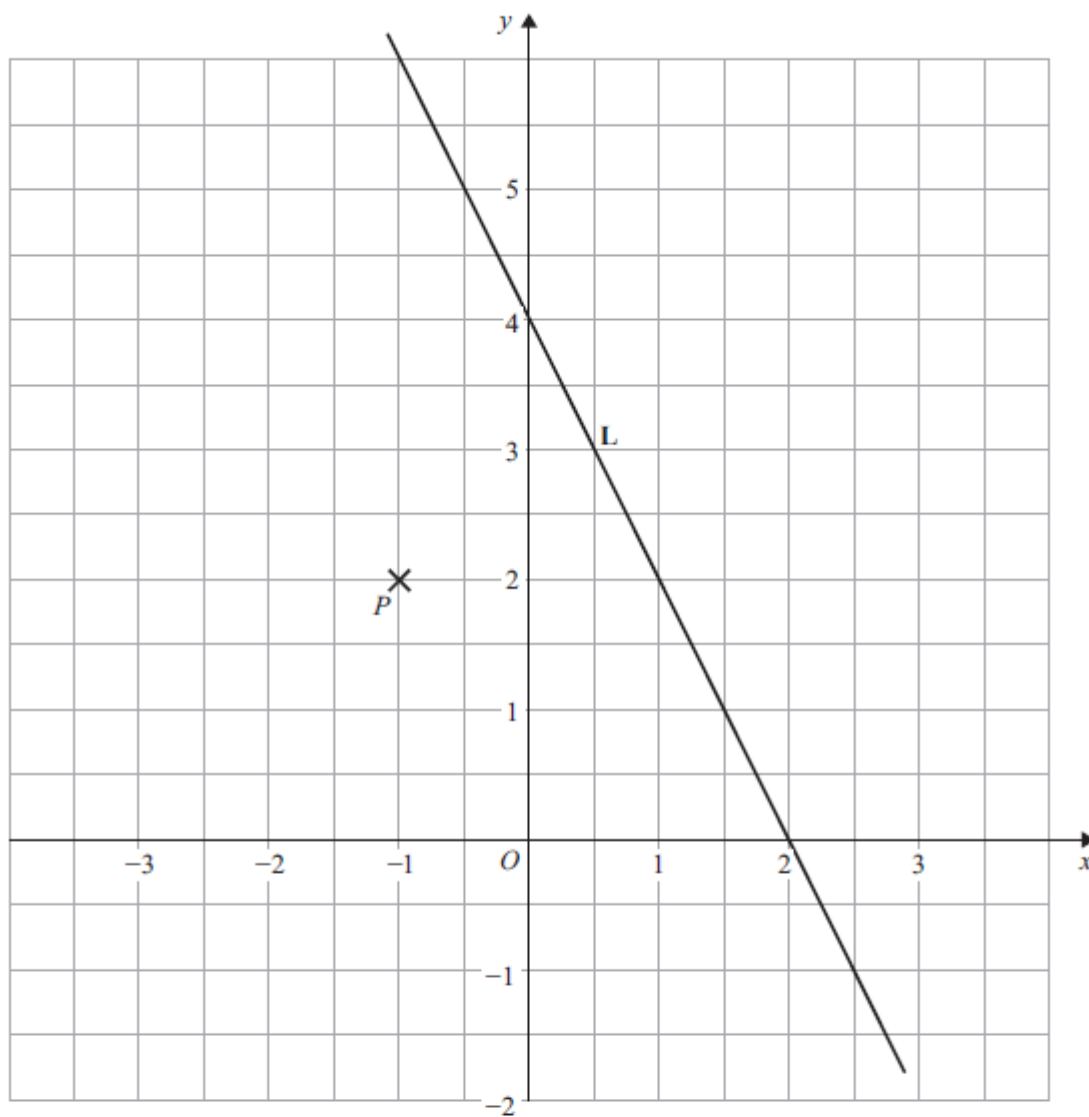
11.

$n$  is an integer greater than 1

Use algebra to show that  $(n^2 - 1) + (n - 1)^2$  is always equal to an even number.

12.

The straight line  $L$  is drawn on the grid.



(a) Find an equation of  $L$ .

-----  
(3)



12 continued

$P$  is the point with coordinates  $(-1, 2)$ .

(b) (i) Find an equation of the straight line that is parallel to  $L$  and passes through  $P$ .

.....  
(ii) Find an equation of the straight line that is perpendicular to  $L$  and passes through  $P$ .

.....  
(4)

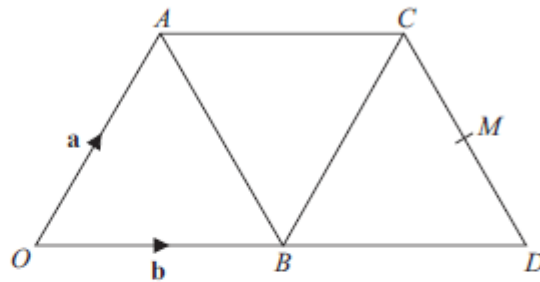
13.

$OACD$  is a trapezium made from three equilateral triangles.

$$\vec{OA} = \mathbf{a}$$

$$\vec{OB} = \mathbf{b}$$

$M$  is the midpoint of  $CD$ .



(a) Write  $\vec{AB}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

.....  
(1)

(b) Show that  $\vec{OC}$  is parallel to  $\vec{BM}$ .

(4)

14.

Simplify  $\frac{4x^2 - 9}{2x^2 - 5x + 3}$

(3 marks)

15.

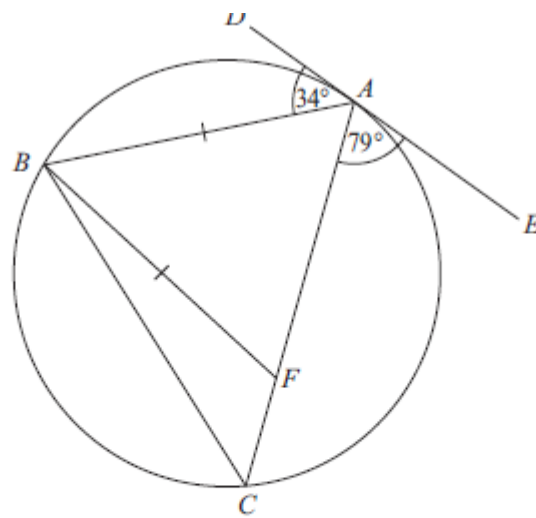


Diagram NOT  
accurately drawn

$A$ ,  $B$  and  $C$  are points on the circumference of a circle.  
 $DAE$  is a tangent to the circle.  
 $F$  is the point on  $AC$  such that triangle  $ABF$  is isosceles.

$$AB = BF$$

$$\text{Angle } DAB = 34^\circ$$

$$\text{Angle } EAC = 79^\circ$$

Work out the size of angle  $CBF$ .

You must give reasons for your answer.

16.

Giving your answers in the form  $a + b\sqrt{2}$ , where  $a$  and  $b$  are rational numbers, find

(a)  $(3 - \sqrt{8})^2$ ,

(3)

(b)  $\frac{1}{4 - \sqrt{8}}$ .

(3)

17.

A bag contains  $(n + 7)$  tennis balls.  
 $n$  of the balls are yellow.  
The other 7 balls are white.

John will take at random a ball from the bag.  
He will look at its colour and then put it back in the bag.

- (a) (i) Write down an expression, in terms of  $n$ , for the probability that John will take a white ball.

.....

Bill states that the probability that John will take a white ball is  $\frac{2}{5}$

- (ii) Prove that Bill's statement cannot be correct.

(3)

After John has put the ball back into the bag, Mary will then take at random a ball from the bag.  
She will note its colour.

- (b) Given that the probability that John and Mary will take balls with **different** colours is  $\frac{4}{9}$ ,  
prove that  $2n^2 - 35n + 98 = 0$

(5)

- (c) Using your previous answer or otherwise, calculate the probability that John and Mary will both take white balls.

.....

(2)