



# Lucton School Mathematics Entrance Paper 2019 -2020

## A level Entry

**Time: 1 hour**  
**Calculator allowed**

**Instructions:**

- Use black ink or ball point pen
- Use pencil for graphs
- Answer all questions
- You may use a calculator
- You may not use a mobile phone or ask for help

**Full Name:**

**Date of Birth:**

**Date of  
Paper:**

International GCSE Mathematics

Formulae sheet – Higher Tier

**Arithmetic series**

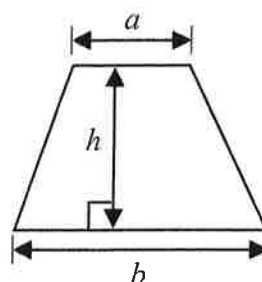
Sum to  $n$  terms,  $S_n = \frac{n}{2} [2a + (n - 1)d]$

**The quadratic equation**

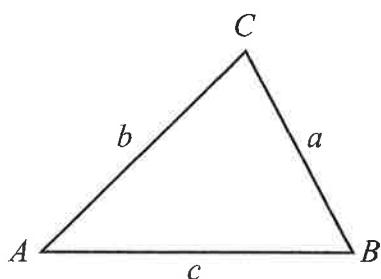
The solutions of  $ax^2 + bx + c = 0$  where  $a \neq 0$  are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Area of trapezium** =  $\frac{1}{2}(a + b)h$



**Trigonometry**



**In any triangle ABC**

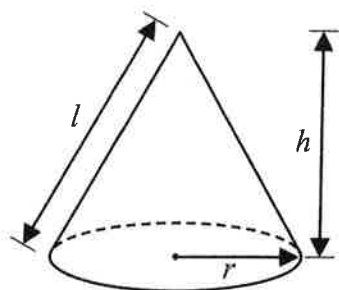
**Sine Rule**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine Rule**  $a^2 = b^2 + c^2 - 2bc \cos A$

**Area of triangle** =  $\frac{1}{2} ab \sin C$

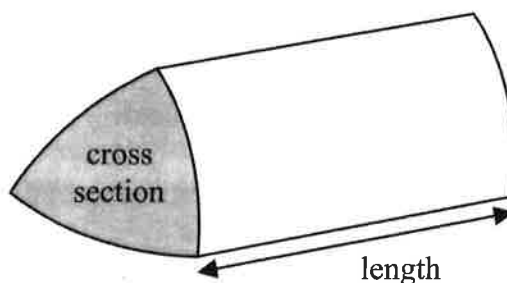
**Volume of cone** =  $\frac{1}{3} \pi r^2 h$

**Curved surface area of cone** =  $\pi r l$



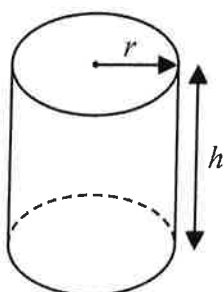
**Volume of prism**

= area of cross section  $\times$  length



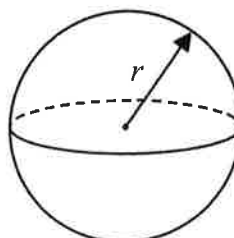
**Volume of cylinder** =  $\pi r^2 h$

**Curved surface area of cylinder** =  $2\pi r h$



**Volume of sphere** =  $\frac{4}{3} \pi r^3$

**Surface area of sphere** =  $4\pi r^2$



Answer all TWENTY questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 The table shows information about the weights, in kg, of 40 parcels.

Weight of parcel ( $p$ kg)	Frequency
$0 < p \leq 1$	19
$1 < p \leq 2$	12
$2 < p \leq 3$	5
$3 < p \leq 4$	2
$4 < p \leq 5$	2

- (a) Write down the modal class.

.....  
(1)

- (b) Work out an estimate for the mean weight of the parcels.

..... kg

(4)

(Total for Question 1 is 5 marks)



2 There are some people in a cinema.

$\frac{3}{5}$  of the people in the cinema are children.

For the children in the cinema,

number of girls : number of boys = 2 : 7

There are 170 girls in the cinema.

Work out the number of adults in the cinema.

(Total for Question 2 is 5 marks)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

3 (a) Simplify  $y^5 \times y^9$

.....  
(1)

(b) Simplify  $(2m^3)^4$

.....  
(2)

(c) Solve  $5(x + 3) = 3x - 4$   
Show clear algebraic working.

$x =$  .....  
(3)

(d) (i) Factorise  $x^2 + 2x - 24$

.....  
(2)

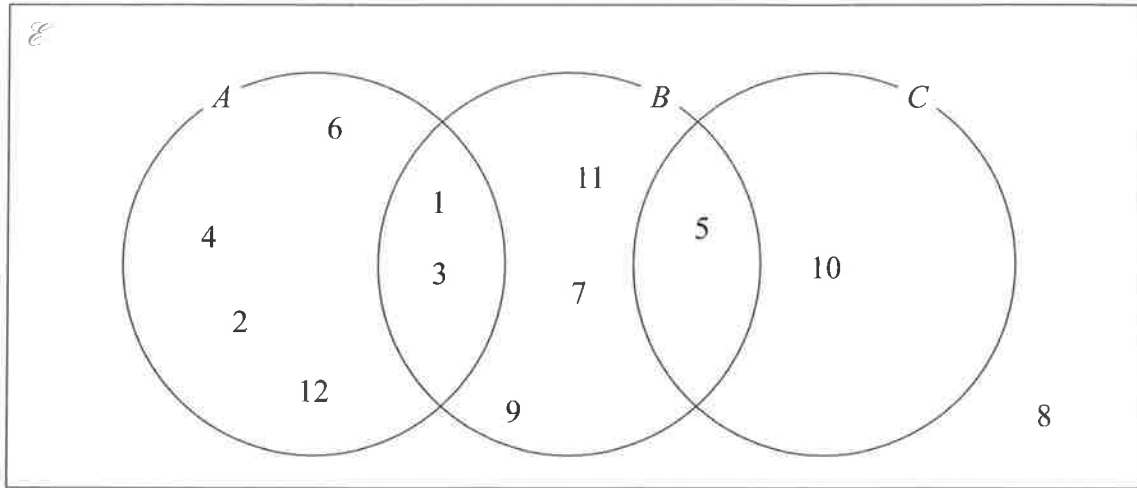
(ii) Hence, solve  $x^2 + 2x - 24 = 0$

.....  
(1)

(Total for Question 3 is 9 marks)



4 Here is a Venn diagram.



(a) Write down the numbers that are in the set

(i)  $A$

.....

(ii)  $B \cup C$

.....

(2)

Brian writes down the statement  $A \cap C = \emptyset$

(b) Is Brian's statement correct?

You must give a reason for your answer.

.....

.....

(1)

One of the numbers in the Venn diagram is picked at random.

(c) Find the probability that this number is in set  $C'$

.....

(2)

(Total for Question 4 is 5 marks)



5 (a) Write  $8 \times 10^4$  as an ordinary number.

.....  
(1)

(b) Work out  $(3.5 \times 10^5) \div (7 \times 10^8)$   
Give your answer in standard form.

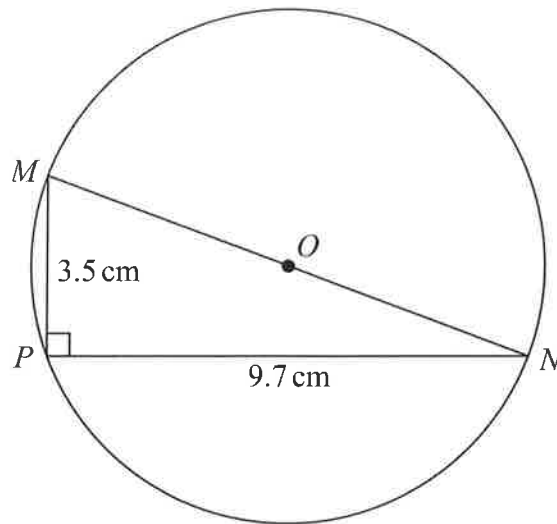
.....  
(2)

**(Total for Question 5 is 3 marks)**



6

Diagram **NOT**  
accurately drawn



$M$ ,  $N$  and  $P$  are points on a circle, centre  $O$ .  
 $MON$  is a diameter of the circle.

$MP = 3.5$  cm  
 $PN = 9.7$  cm

Angle  $MPN = 90^\circ$

Work out the circumference of the circle.  
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 6 is 4 marks)





DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

7 Chao bought a boat for HK\$160 000  
The value of the boat depreciates by 4% each year.

- (a) Work out the value of the boat at the end of 3 years.  
Give your answer correct to the nearest HK\$.

HK\$.....  
(3)

Jalina gets a salary increase of 5%  
Her salary after the increase is HK\$252 000

- (b) Work out Jalina's salary before the increase.

HK\$.....  
(3)

(Total for Question 7 is 6 marks)



8  $A = 3^5 \times 5 \times 7^3$   
 $B = 2^3 \times 3 \times 7^4$

(a) (i) Find the Highest Common Factor (HCF) of  $A$  and  $B$ .

(ii) Find the Lowest Common Multiple (LCM) of  $A$  and  $B$ .

(2)

$A = 3^5 \times 5 \times 7^3$   
 $B = 2^3 \times 3 \times 7^4$   
 $C = 2^p \times 5^q \times 7^r$

Given that

the HCF of  $B$  and  $C$  is  $2^3 \times 7$

the LCM of  $A$  and  $C$  is  $2^4 \times 3^5 \times 5^2 \times 7^3$

(b) find the value of  $p$ , the value of  $q$  and the value of  $r$ .

$p =$  .....

$q =$  .....

$r =$  .....

(2)

(Total for Question 8 is 4 marks)



9 The diagram shows a right-angled triangle.

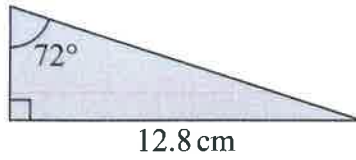


Diagram NOT accurately drawn

Five of these triangles are put together to make a shape.

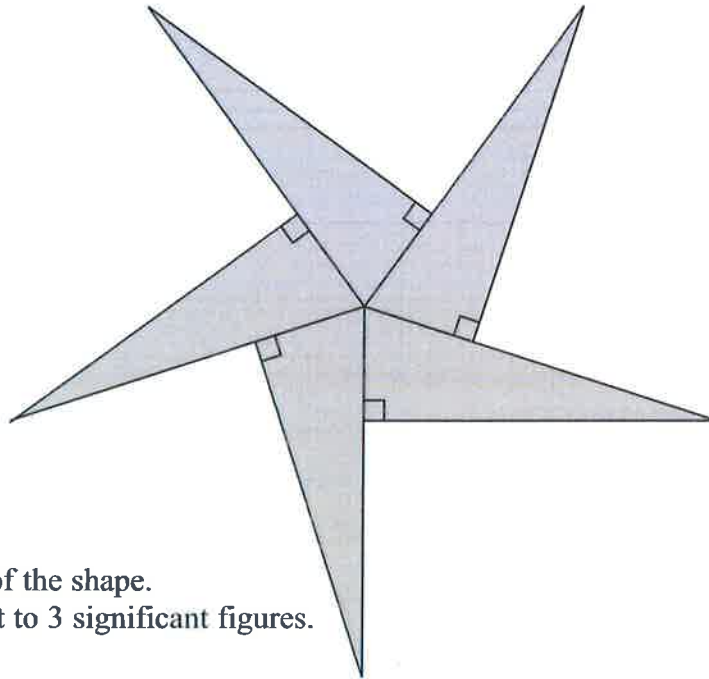


Diagram NOT accurately drawn

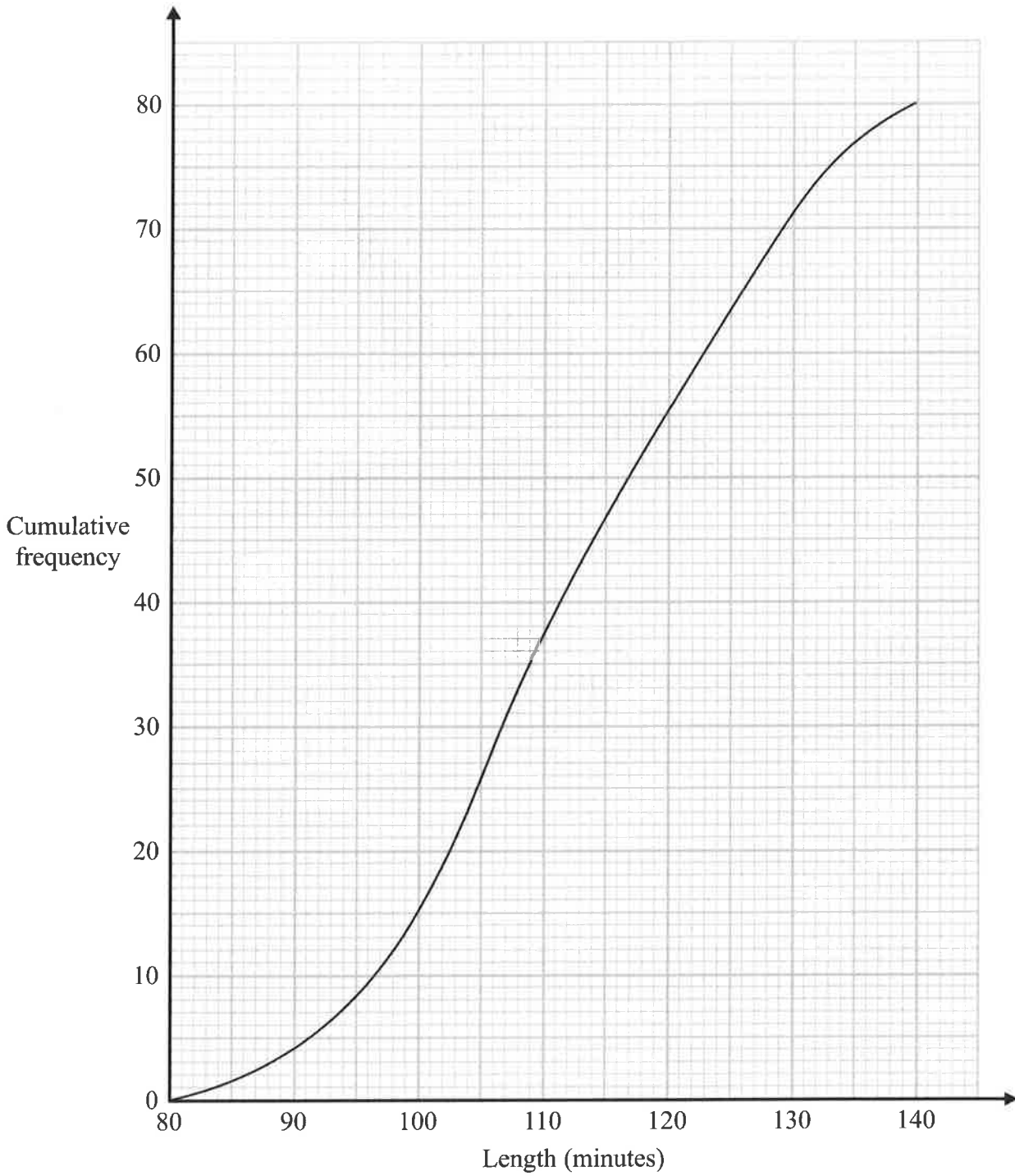
Calculate the perimeter of the shape.  
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 9 is 5 marks)



10 The cumulative frequency graph shows information about the length, in minutes, of each of 80 films.



(a) Use the graph to find an estimate for the interquartile range.

..... minutes  
(2)



Clare says,

“More than 35% of these films are over 120 minutes long.”

(b) Is Clare correct?

Give a reason for your answer.

(3)

**(Total for Question 10 is 5 marks)**



11 (a) Expand and simplify  $(2x - 1)(x + 3)(x - 5)$

---

(3)

(b) Solve  $3x^2 + 6x - 5 = 0$   
Show your working clearly.  
Give your solutions correct to 3 significant figures.

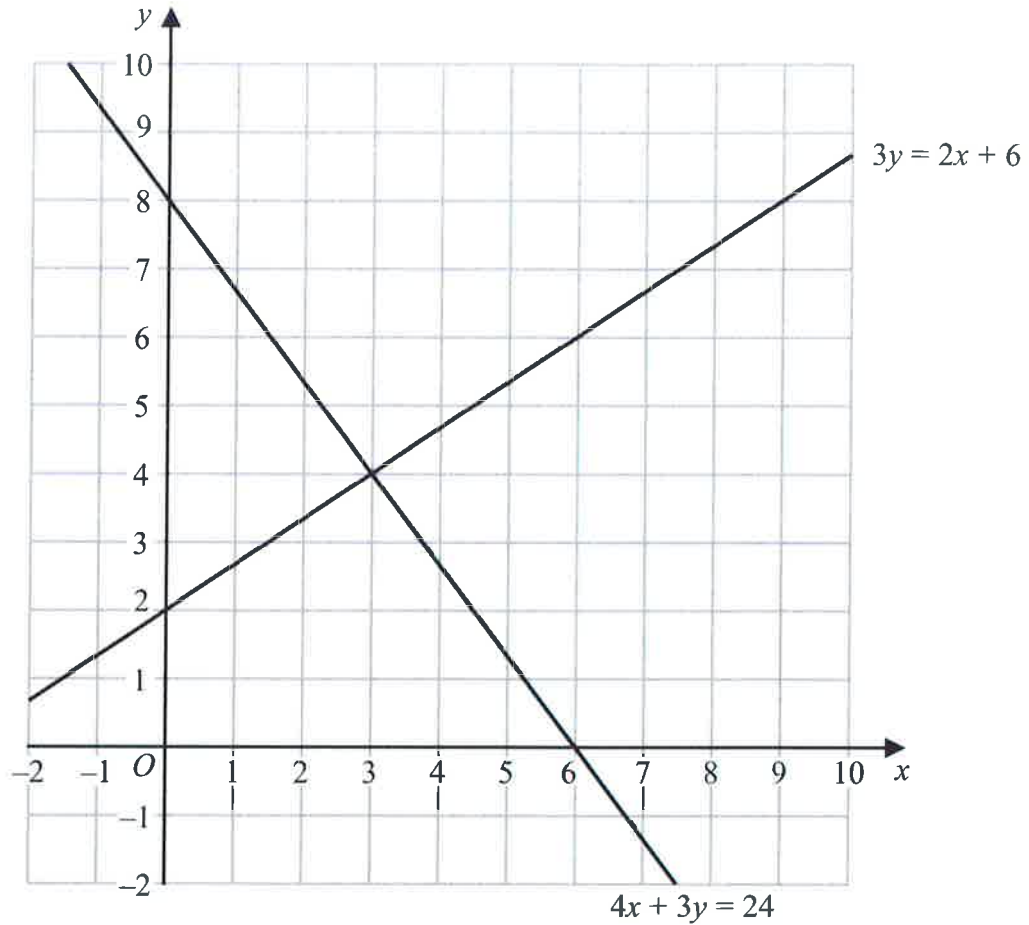
---

(3)

**(Total for Question 11 is 6 marks)**



12 The diagram shows two straight lines drawn on a grid.



(a) Write down the solution of the simultaneous equations

$$\begin{aligned} 3y &= 2x + 6 \\ 4x + 3y &= 24 \end{aligned}$$

$x =$  .....

$y =$  .....

(1)

(b) Show, by shading on the grid, the region defined by all five of the inequalities

$$x \geq 0 \quad y \geq 0 \quad x + y \geq 4 \quad 3y \leq 2x + 6 \quad 4x + 3y \leq 24$$

Label the region R.

(3)

(Total for Question 12 is 4 marks)



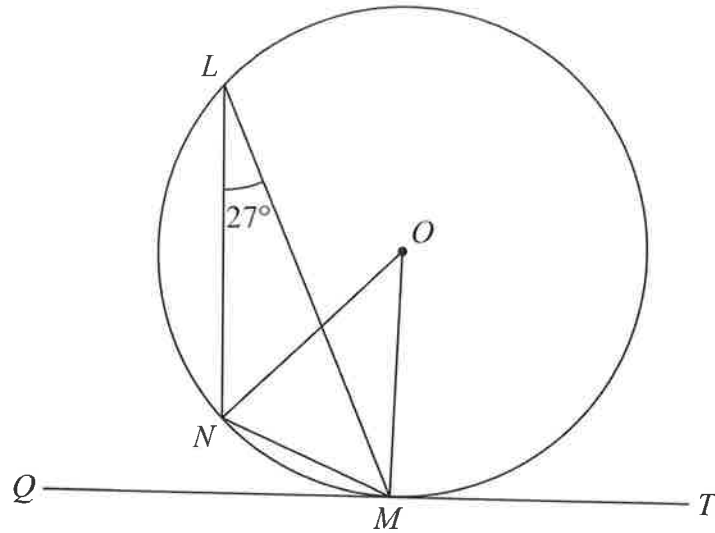


Diagram **NOT** accurately drawn

$L$ ,  $M$  and  $N$  are points on a circle, centre  $O$ .  
 $QMT$  is the tangent to the circle at  $M$ .

(a) (i) Find the size of angle  $NOM$ .

..... 0

(ii) Give a reason for your answer.

.....  
 .....  
 (2)

(b) (i) Find the size of angle  $NMQ$ .

..... 0

(ii) Give a reason for your answer.

.....  
 .....  
 (2)

(Total for Question 13 is 4 marks)





14 The function  $f$  is such that

$$f(x) = \frac{3x - 5}{4}$$

(a) Find  $f(-7)$

.....  
(1)

(b) Express the inverse function  $f^{-1}$  in the form  $f^{-1}(x) = \dots$

$$f^{-1}(x) = \dots$$

(2)

The function  $g$  is such that

$$g(x) = \sqrt{19 - x}$$

(c) Find  $fg(3)$

.....  
(2)

(d) Which values of  $x$  cannot be included in any domain of  $g$ ?

.....  
(2)

(Total for Question 14 is 7 marks)



15 (a) Simplify fully  $\left(\frac{256x^{20}}{y^8}\right)^{-\frac{1}{4}}$

(2)

(b) Express  $\frac{1}{9x^2 - 25} - \frac{1}{6x + 10}$  as a single fraction in its simplest form.

(3)

(Total for Question 15 is 5 marks)



DO NOT WRITE IN THIS AREA

16 A frustum is made by removing a small cone from a large cone. The cones are mathematically similar.

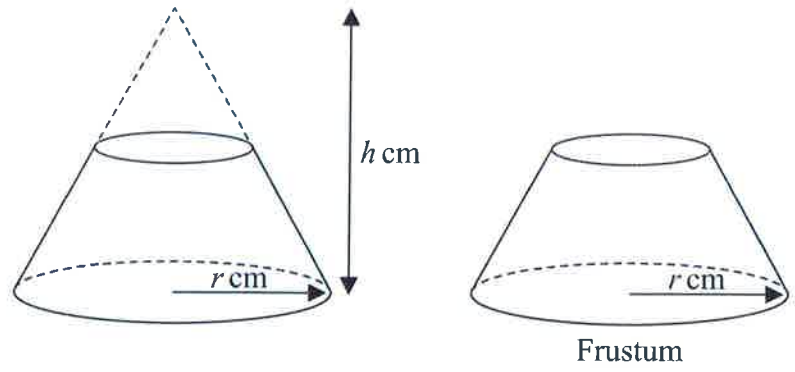


Diagram NOT accurately drawn

The large cone has base radius  $r$  cm and height  $h$  cm.

Given that

$$\frac{\text{volume of frustum}}{\text{volume of large cone}} = \frac{98}{125}$$

find an expression, in terms of  $h$ , for the height of the frustum.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

..... cm

(Total for Question 16 is 4 marks)



17 The diagram shows parallelogram  $ABCD$ .

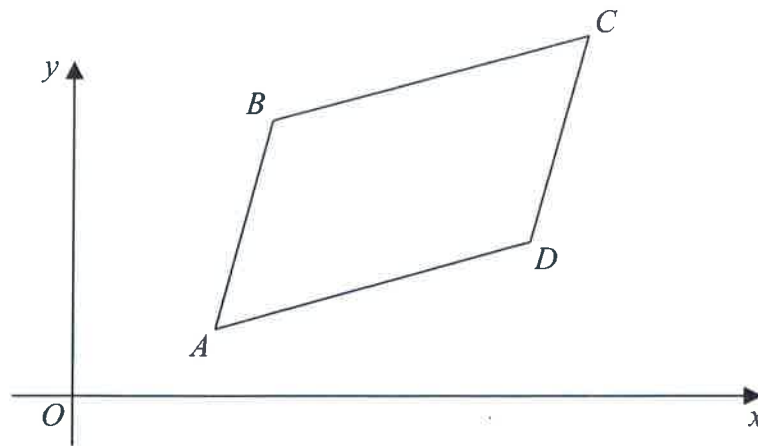


Diagram **NOT** accurately drawn

$$\vec{AB} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} 10 \\ 11 \end{pmatrix}$$

The point  $B$  has coordinates  $(5, 8)$

(a) Work out the coordinates of the point  $C$ .

(....., .....)  
(3)

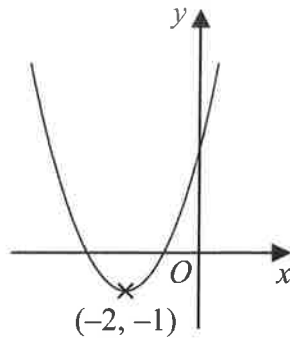
The point  $E$  has coordinates  $(63, 211)$

(b) Use a vector method to prove that  $ABE$  is a straight line.

(2)

(Total for Question 17 is 5 marks)





The diagram shows the curve with equation  $y = f(x)$

The coordinates of the minimum point of the curve are  $(-2, -1)$

(a) Write down the coordinates of the minimum point of the curve with equation

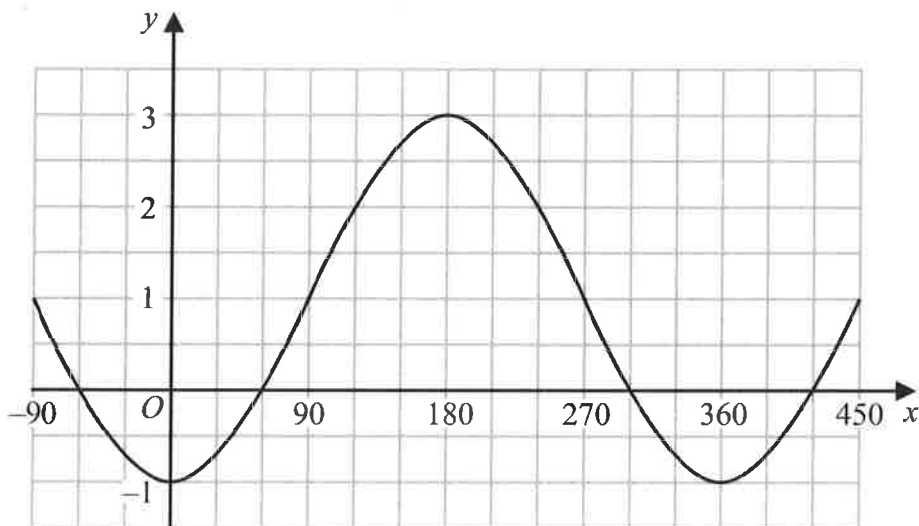
(i)  $y = f(x - 5)$

(....., .....) )

(ii)  $y = \frac{1}{2}f(x)$

(....., .....) )  
(2)

The graph of  $y = a \sin(x - b)^\circ + c$  for  $-90 \leq x \leq 450$  is drawn on the grid below.



(b) Find the value of  $a$ , the value of  $b$  and the value of  $c$ .

$a =$  .....

$b =$  .....

$c =$  .....

(3)

(Total for Question 18 is 5 marks)

19 Jack plays a game with two fair spinners, **A** and **B**.

Spinner **A** can land on the number 2 or 3 or 5 or 7

Spinner **B** can land on the number 2 or 3 or 4 or 5 or 6

Jack spins both spinners.

He wins the game if one spinner lands on an odd number **and** the other spinner lands on an even number.

Jack plays the game twice.

Work out the probability that Jack wins the game both times.

(Total for Question 19 is 4 marks)



20  $ABC$  is an isosceles triangle such that

$$AB = AC$$

$A$  has coordinates  $(4, 37)$

$B$  and  $C$  lie on the line with equation  $3y = 2x + 12$

Find an equation of the line of symmetry of triangle  $ABC$ .

Give your answer in the form  $px + qy = r$  where  $p$ ,  $q$  and  $r$  are integers.

Show clear algebraic working.

(Total for Question 20 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS



**BLANK PAGE**

